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La Taptana

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The *taptana* is a contemporary version of a historical abacus device known as the "Cañari Counter" ("contador Cañari") from Ecuador. Numerous finds of this kind in the region of Cañar suggest that many years ago rocks with inscriptions used for counting purposes were commonplace; thus, the historical Ecuadorian abacus carries the name of the region in which it was found. Evidence of such carved rocks is found in museums throughout Ecuador (Fig. 1), although it is difficult to acquire images of such ancient artifacts. Two such devices were referenced by Carlos Radicati di Primeglio (1990) (Fig. 2), noting that these particular rock counters found in Ecuador are different from the *yupanas* of Peru because the counting compartments are created by incisions on the surface of the rock instead of carved hollows. These devices also differ from the yupanas because their composition is more indicative of a circular counting motion as noted by Montaluisa (Fig. 3).

The historical abacus, the contador Cañari, is a tablet composed of two three-by-three tables located diagonally across from one another, with additional spaces carved alongside (Fig. 3). The tables are used to move stones in a circular motion to either add or subtract values, and the five spaces on either side of these tables could have represented the first five multiples of 10: 10, 20, 30, 40, and 50. The presence of five holes on either side of both of the table incisions does not appear to be coincidental as many Ecuadorian languages favor multiples of 5 in their linguistic understanding and labeling of numeric values (Fig. 4). This interpretation would only work for adding and subtracting values whose result did not supersede 59; it is probable that alternative methods were used for representing and manipulating larger values.

Using the circular motion and tabular values proposed by Montaluisa (Fig. 3), we propose a method of addition and subtraction whose simultaneous circular motions act as a dual system of checks and balances, typical of Andean math systems and records, as noted by Gary Urton in regard to the *quipu* (2009). The summands are placed on the top and bottom tables, and the bottom summand is added to the top summand by moving the top summand in an increasing direction and the bottom summand in a decreasing direction, until the bottom summand reaches 0 and the addition is concluded. As the top summand moves from 9 to 1, a step is taken to increase the tens counter, and as the bottom summand moves from 1 to 9, a step is taken to decrease the tens counter. For example, let us add 27 + 14 by placing 27 on the upper table and 14 on the lower table (Fig. 5:A). We will eventually move the stone in the top table 11 times, passing through the values representing 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, and 41 while the bottom counter is reduced to 0 (Fig. 5:B–O). The final summation, 41, is represented on the final taptana (Fig. 5:O). Additional tokens used in the calculation could be placed in the carved hollow above the tables, which has been eliminated from this sketch for the sake of simplicity.

The ingenuity of these historical instruments inspired Luis Montaluisa to create a contemporary version of the abacus for use in education. Montaluisa is a linguist who has dedicated a part of his life to uncovering and promoting Quichua language and cultural production in Ecuador through curriculum development and the creation of local linguistic and numeric models. Montaluisa

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Fig. 1 Ancestral Cañari Taptana. Courtesy of the reserve of the Museum of the Central Bank of Ecuador (Photo by David Montaluisa Álvarez y María Belén Montaluisa Álvarez)

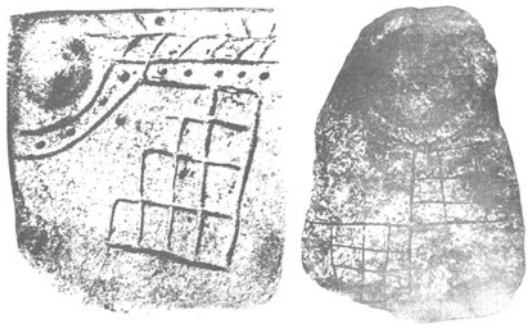

Fig. 2 Historical taptanas (After Radicati di Primeglio, 225)

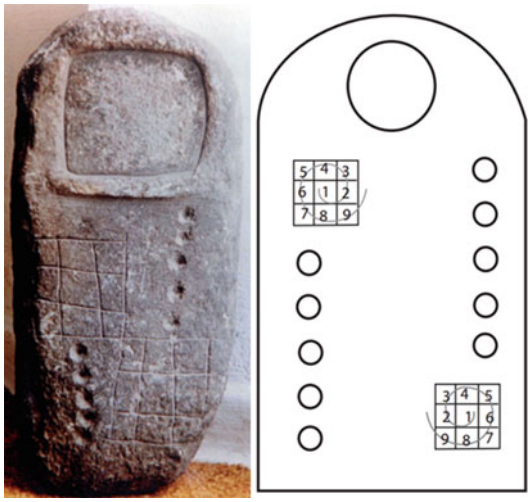

Fig. 3 The Cañari Counters (Contadores Cañari). On the right side we see the values that each square held and a direction used for counting as proposed by Montaluisa (From Montaluisa, 36–37)

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No.	Cha'palaa (Chafiki)	Waotededo	Siapedee	Paikoka/Baikoka
1	mallu	adoke	aba	te ' o / te ' e
2	pallu	mea	omé	kayayë
3	pema	mea go adoke (2+1)	ompé	toasoñë
4	Taapallu (2+2), (2x2)	mea go mea (2+2)	khimari	kajese 'ë
5	manda	emenpoke (right hand)	joisomá	te ' ejënte
6	manchis mallu (5+1)	emenpoke go adoke (5+1)	joisomá aba (5+1)	te ' ejënte te ' o (5+1)
7	manchis pallu (5+2)	emenpoke go mea (5+2)	joisomá omé (5+2)	te ' ejënte kayayë (5+2)
8	manchis pema (5+3)	emenpoke mea go adoke [5+(2+1)]	joisomá ompé (5+3)	te ' ejënte toasoñë (5+3)
9	manchis taapallu [5+(2+2)]	emenpoke mea go mea [5+(2+2)]	joisomá khimari (5+4)	te ' ejënte kajese 'ë (5+4)
10	paytya (2x5)	tipenpoke (left hand)	omé joisomá (2x5)	si ' ajëna

Fig. 4 Numbers 1–10 in various indigenous languages of Ecuador (Image by Molly Tun after Montaluisa, 14–16)

stresses that the Quichua language actually better represents and structures numeric words than many romance languages. While English requires learners to memorize the number lexemes 1 through 19 as these are unique denominators that do not show repetitions in the creation of subsequent values, a Quichua speaker simply needs to memorize the lexemes for 1 through 10 (*shuk*, *ishkay*, *kimsa*, *chuska*, *pichka*, *sukta*, *kanchis*, *pusak*, *iskun*, *chunka*) as every subsequent number is a direct composition of these 10. In addition, English requires the rote memorization of the distinct powers of 10 while in Quichua these powers of ten can be derived from the base number lexemes (ex., $20 = 2 \times 10$ or *ishkay chunka*). This is illustrated below (Fig. 6) in a table similar to the one constructed by Montaluisa (2011), but differentiates between those numbers that can be derived from unit number words and those whose particularities simply require them to be stored in memory. Given the capacity of the Quichua language to represent and understand number patterns in base 10, the taptana actually operates most perfectly when using this language.

The taptana created by Montaluisa, known as the Taptana Montaluisa, highlights this relation by illustrating each power of 10 in a separate column, starting with the ones column on the far right (Fig. 7). Each column is composed of nine holes, indicative of the nine memorized numbers in Quichua (numbers 1–9), and each subsequent column presents a different multiplier of 10 (*chunka*). This abacus not only shares a close relationship with the Quichua language but also with the representation of numbers using Arabic numerals, common to most mathematics classrooms. In a fashion similar to that of adding with pen and paper, the taptana can be used to add, subtract, multiply (repeated addition), and divide (repeated subtraction), by combining the tokens in the units column first and working from right to left. Only two rules regulate taptana calculations: (1) groups

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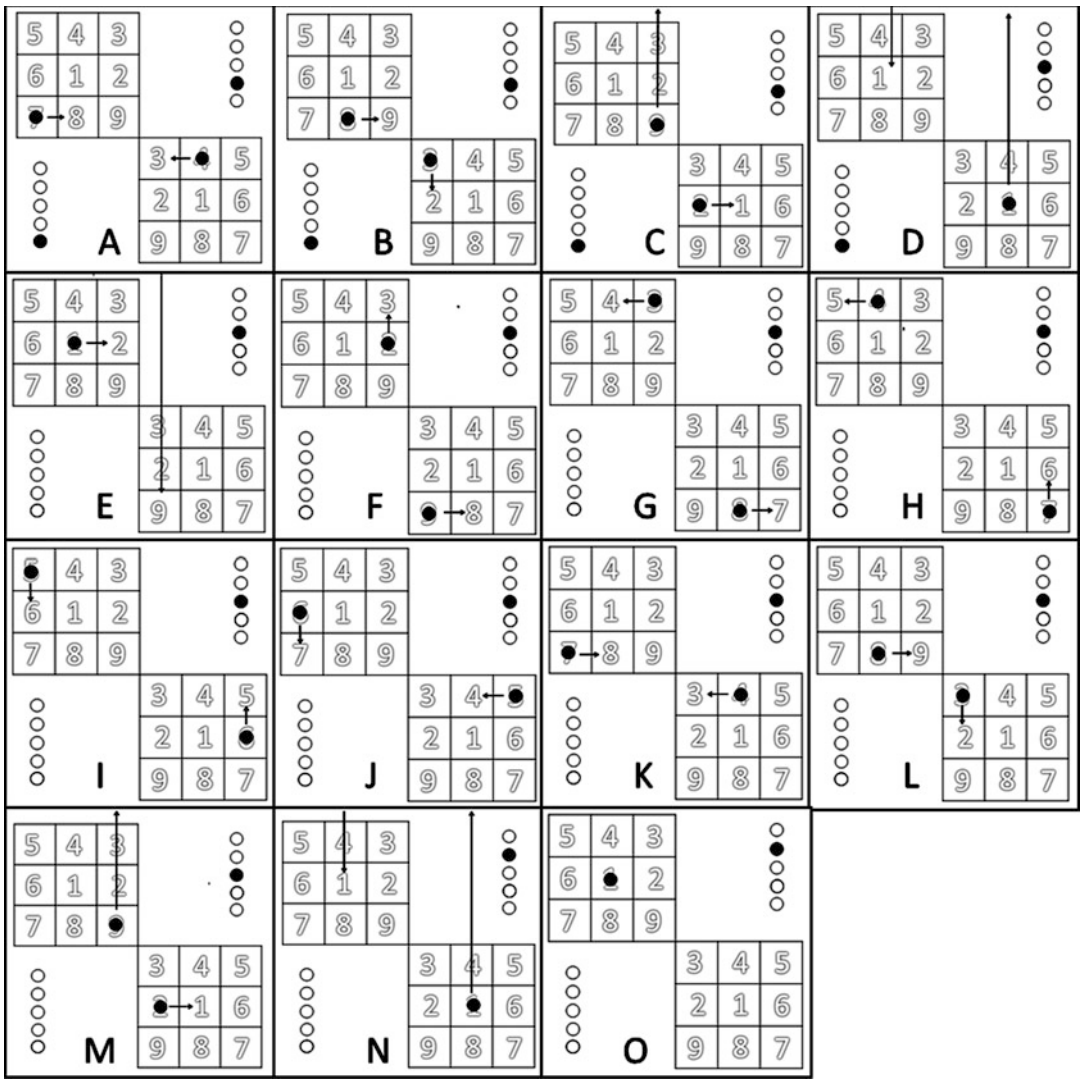

Fig. 5 Possible method of addition on the Cañari Counter, showing 27 + 14 = 41 (Image by Molly Tun)

of 10 tokens are bundled together in a single token, and (2) bundled tokens are placed in the adjoining column on the left.

This calculating device is also known as the Taptana Nikichik which is identical except for the implementation of distinct colors in each column; the taptana ultimately designed by Montaluisa has been copied and reproduced by other individuals and organizations claiming it as their own. Those parties benefitting from taptana designs along with the attention this calculating device has received in contemporary society and its widespread use have led Montaluisa to patent his design, although he does wish for greater diffusion of the taptana to aid in mathematics education efforts throughout Ecuador (Fig. 8). Montaluisa named this counting device the "taptana" because this was the denomination given to the "chess" or "playing boards" in Domingo de Santo Tomás's Quechua dictionary *Lexicon*, *o vocabulario de la lengua general de Peru* (1560). Montaluisa also notes that Nikichik is misleading as this term in Quechua refers to ordinal numbers as opposed to cardinal numbers. He writes: "I must inform that some people, without consent of the author, have started denominating the Taptana Montaluisa as Taptana Nikichik and place different colors in the columns.

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No.	Quichua	English	Spanish
1	Shuk	One	Uno
2	Ishkay	Two	Dos
3	Kimsa	Three	Tres
4	Chusku	Four	Cuatro
5	Pichka	Five	Cinco
6	Sukta	Six	Seis
7	Kanchis	Seven	Siete
8	Pusak	Eight	Ocho
9	Iskun	Nine	Nueve
10	Chunka	Ten	Diez
11	Chunka Shuk (10+1)	Eleven	Once
12	Chunka Ishkay (10+2)	Twelve	Doce
13	Chunka Kimsa (10+3)	Thirteen	Trece
14	Chunka Chusku (10+4)	Fourteen (4+10?)	Catorce
15	Chunka Pichka (10+5)	Fifteen	Quince
16	Chunka Sukta (10+6)	Sixteen (6+10?)	Dieciseis (10+6)
17	Chunka Kanchis (10+7)	Seventeen (7+10?)	Diecisiete (10+7)
18	Chunka Pusac (10+8)	Eighteen (8+10?)	Dieceocho (10+8)
19	Chunka Iskun (10+9)	Nineteen (9+10?)	Diecinueve (10+9)
20	Ishkay Chunka (2x10)	Twenty	Veinte
30	Kimsa Chunka (3x10)	Thirty	Treinta
40	Chuska Chunka (4x10)	Forty	Cuarenta
50	Pichka Chunka (5x10)	Fifty	Cincuenta
60	Sukta Chunka (6x10)	Sixty (6x10?)	Sesenta
70	Kanchis Chunka (7x10)	Seventy (7x10?)	Setenta
80	Pusak Chunka (8x10)	Eighty (8x10?)	Ochenta
90	Iskun Chunka (9x10)	Ninety (9x10?)	Noventa
99	Iskun Chunka Iskun (9x10+9)	Ninety nine (9x10?+9)	Noventa y nueve (90+9)

Fig. 6 Table of number lexemes in Quichua, English, and Spanish. Numeration in Quichua requires far less memorization of number lexemes than in other languages in order to derive the full range of values (Image by Molly Tun after Montaluisa, 12)

This is a mathematical error, since *nikina* in Quechua, is to order a sequence in relation to a referent. *Nikina*, from which come the words *niki*, *nikichina*, *nikichik*, serve to designate the ordinal numbers, not to represent the cardinal numbers. Nor does it explain a system of positional enumeration. The *niki* is used for example in *Shukniki* (first, this appears to come from *shuk mikpi*), *ishkayniki* (second), *kimsaniki* (third), etc. *Nikichina* means to order. This doesn't have anything to do with the fundamental objective of the Taptana Montaluisa, the latter being positional. Therefore, in the Taptana Montaluisa distinct colors are not placed in the columns. Different colors in the columns of

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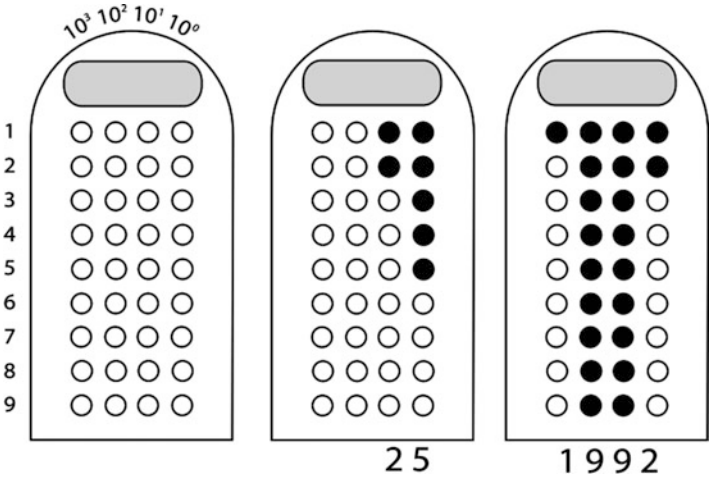

Fig. 7 The Taptana Montaluisa. The columns represent distinct powers of ten, with the ones, tens, hundreds, thousands, etc. moving from left to right. The counters can either be placed closest to the top or bottom of the tablet (Image by Montaluisa, 21)

Fig. 8 Children from Cañar calculating with the Taptana Montaluisa and the Taptana Cañari, 2014 (Photo by Magdalena Guamán)

the taptana are distractors that can cause difficulty in understanding the concept of the positional value of numeric symbols, and of the semiotic process of the formation of abstract thinking and mathematic thought" (2011).

Other variations of the taptana have also been identified in historical and contemporary contexts. As the multitude of indigenous Andean languages have different ways of representing numeric values (see Fig. 4) and rely on different bases, the Taptana Montaluisa could easily be modified to match a numeric system of any base. Just as the base-10 taptana (Fig. 7) has nine spaces in each column, a base 8 system would contain seven spaces, a base 5 system would contain four spaces, etc. Other circular counting tablets (Fig. 9) have been designed such as that used by Yonit Bousany in a bilingual Quechua-Spanish school outside of Cusco (2008). With this method a summand is placed on the table and another summand may be added by moving the units token in a counter-clockwise motion the desired number of additional spaces. The only rule to this taptana is that whenever a token crosses 0, the next larger place value token must be increased one space. This variation of the taptana uses stones of various sizes (or colors) to represent the ones, tens, and hundreds. This method reflects the various Andean traditions of using stones (or knots) of multiple sizes as indicators of place value in order to calculate (Párraga Chirveches 1992; Figs. 10–11).

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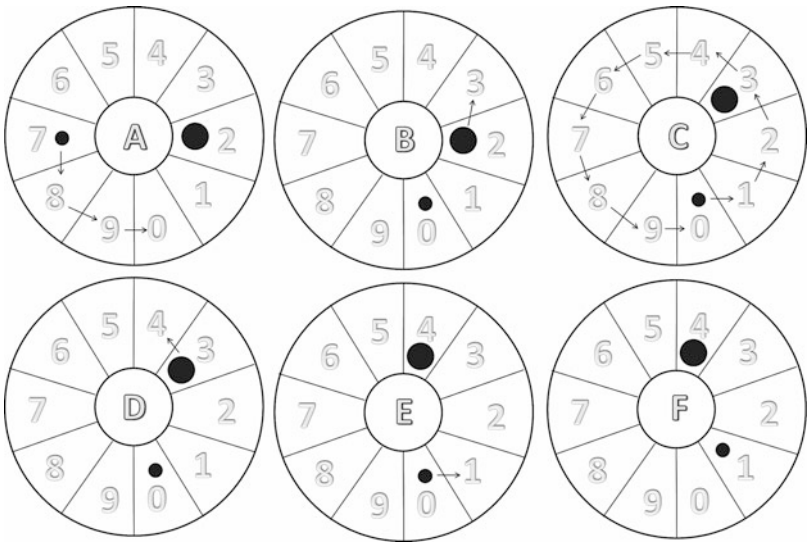

Fig. 9 Adding on a circular version of the taptana, showing 27 + 14 = 41 (Image by Molly Tun)

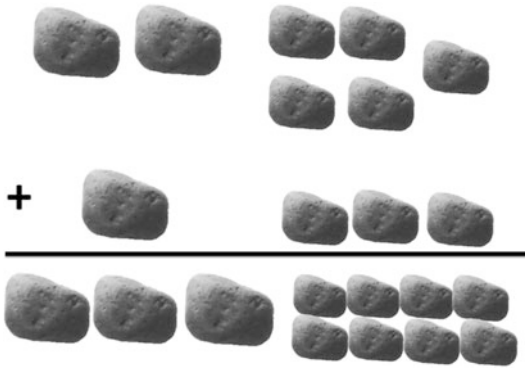

Fig. 10 This image shows the calculation of oxen prices (Bs 2.500 + Bs 1.300 = Bs 3.800) as practiced in the Bolivian provinces of Camacho, Loayza, and Ingavil (Image by Jennifer Leonard)

The studies working to promote knowledge and use of both the contemporary taptana and the historical Cañari Counter contribute to an ethnomathematical discussion of alternative conceptions of numeracy and quantification in an attempt to recover indigenous ways of knowing. The circular counting techniques of Ecuador and other Andean communities deeply contrast with the linear, chronological number line as conceived of in Western culture and provide definite advantages to the numeration and manipulation of values. Taptana is oftentimes associated with indigenous board games (Arriaga, Verneau, and Rivet) and has been equated to the game shown in Guaman Poma de Ayala's illustration (Holm 1958; Fig. 12). Regardless, the knowledge of these game boards and counting tablets was lost with the extermination of indigenous culture, in this case symbolically represented through the execution of Atahualpa (Fig. 12). Although many of the exact details regarding the historic counting techniques of the Andes are unknown, engaging in the tradition of the taptana seeks to recover the indigenous perspective that has been obscured through processes of colonization and globalization.

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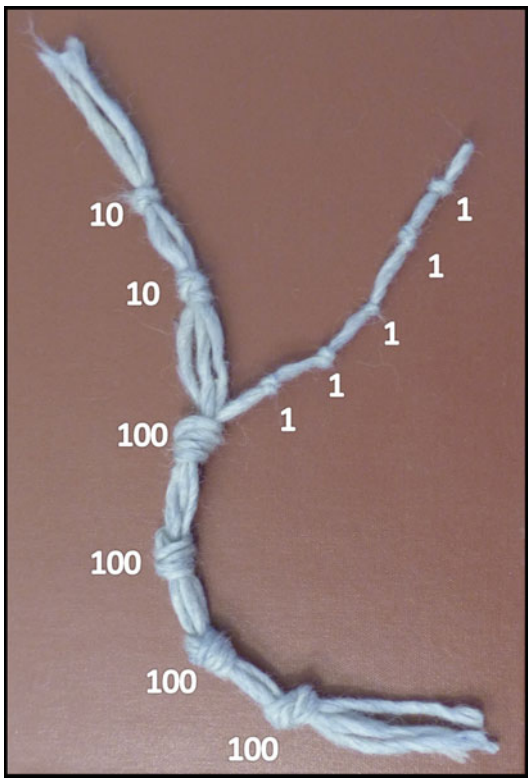

Fig. 11 In the Bolivian province of Omasuyos different sizes of knots are used to represent different place values of ten as seen in the knotted value of 425 above (Photo by Molly Tun)

Fig. 12 Atahualpa and a Spanish soldier gathered around a taptana or alquerque game board before Atahualpa's execution

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